



Unit 2 Revision Sheet C Function Notation Transformation of Graphs Differentiation Higher

Questions

Q1.

The functions f and g are such that

$$f(x) = 2x - 3$$

$$g(x) = \frac{x}{3x + 1}$$

(a) State the value of x that cannot be included in any domain of g

.....

(1)

(b) Find $gf(x)$

Simplify your answer.

$$gf(x) = \dots\dots\dots$$

(2)

(c) Express the inverse function g^{-1} in the form $g^{-1}(x) = \dots$

$$g^{-1}(x) = \dots\dots\dots$$

(3)

(Total for question = 6 marks)



Q2.

The functions f and g are defined as

$$f(x) = 5x^2 - 10x + 7 \quad \text{where } x \geq 1$$

$$g(x) = 7x - 6$$

(a) Find $fg(2)$

.....

(2)

(b) Express the inverse function f^{-1} in the form $f^{-1}(x) = \dots$

$$f^{-1}(x) = \dots\dots\dots$$

(4)

(Total for question = 6 marks)



Q3.

g is the function with domain $x \geq -3$ such that $g(x) = x^2 + 6x$

(a) Write down the range of g^{-1}

(1)

(b) Express the inverse function g^{-1} in the form $g^{-1} : x \mapsto \dots$

$g^{-1} : x \mapsto$

(4)

(Total for question = 5 marks)

Q4.

f is the function such that $f(x) = 4 - 3x$

(a) Work out $f(5)$

(1)

g is the function such that $g(x) = \frac{1}{1-2x}$



(b) Find the value of x that cannot be included in any domain of g

(1)

(c) Work out $fg(-1.5)$

(2)

(Total for question = 4 marks)

Q5.

The function f is such that $f(x) = 3x - 2$

(a) Find $f(5)$

.....

(1)

The function g is such that $g(x) = 2x^2 - 20x + 9$ where $x \geq 5$

(b) Express the inverse function g^{-1} in the form $g^{-1}(x) = \dots$

$g^{-1}(x) = \dots$

(4)

(Total for question = 5 marks)



Q6.

The function f is such that $f(x) = x^2 - 8x + 5$ where $x \leq 4$

Express the inverse function f^{-1} in the form $f^{-1}(x) = \dots$

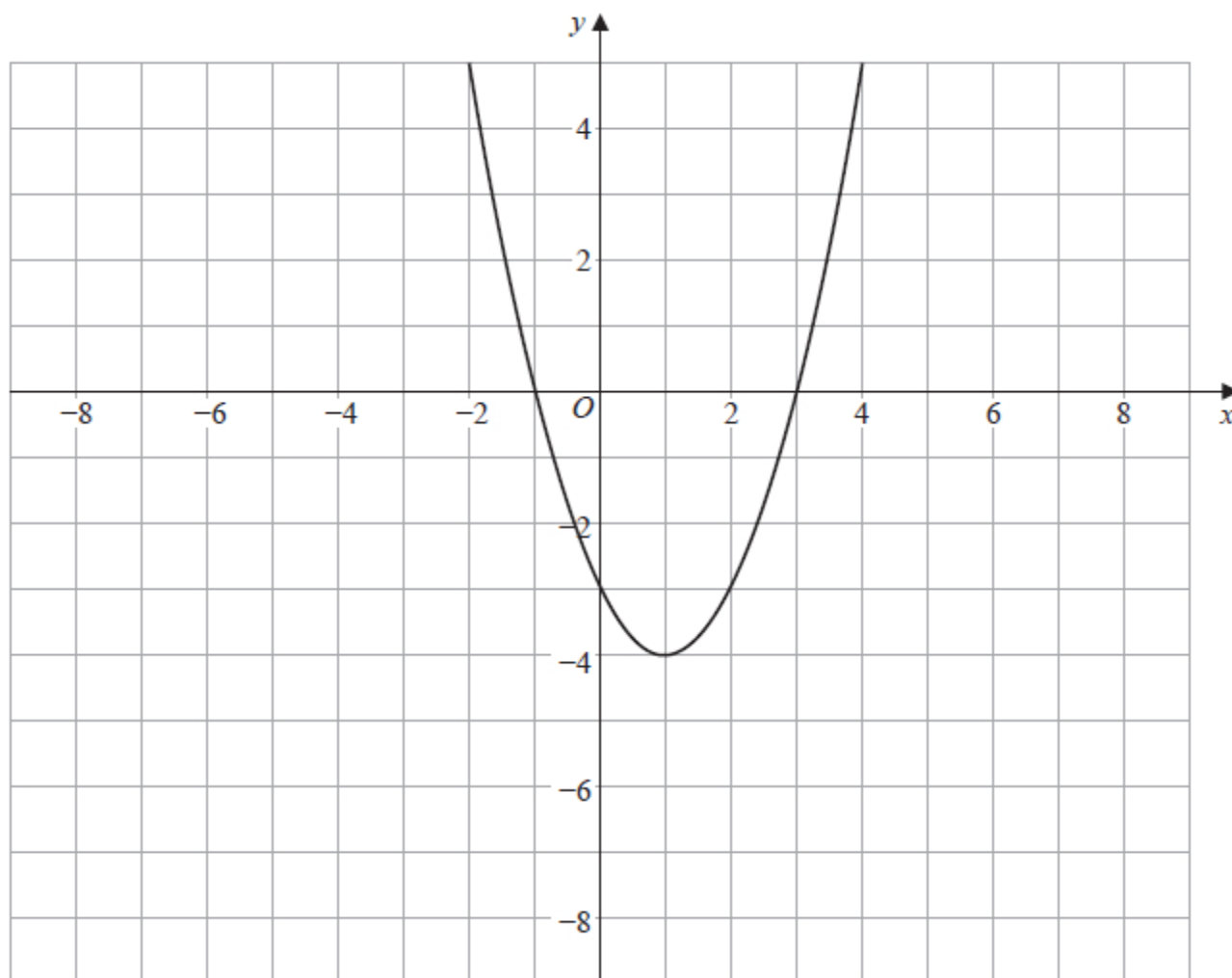
$f^{-1}(x) = \dots\dots\dots$

(Total for question = 3 marks)



Q7.

The graph of $y = f(x)$ is shown on the grid.

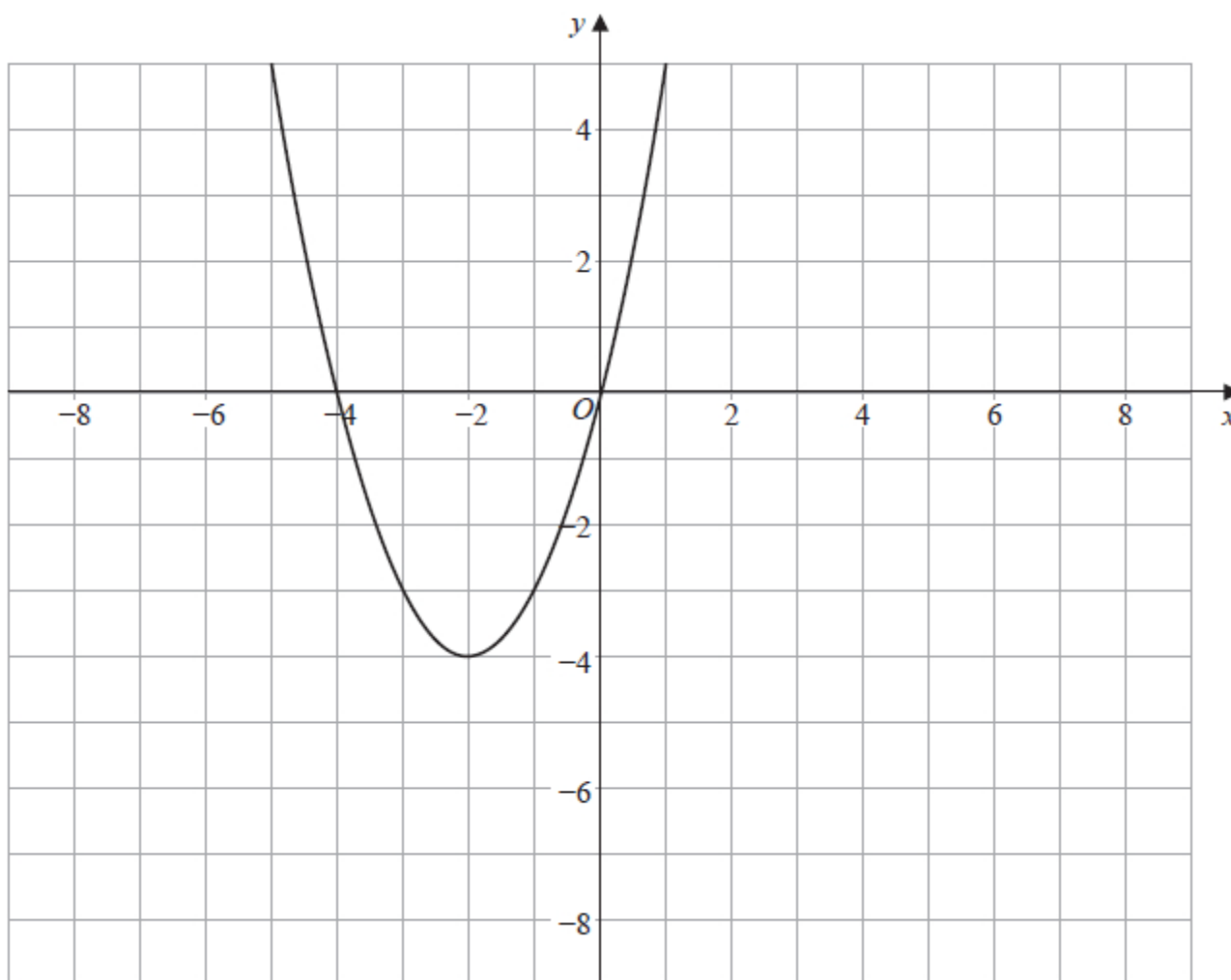


(a) On the grid above, sketch the graph of $y = f\left(\frac{1}{2}x\right)$

(2)



The graph of $y = f(x + k)$ is shown on the grid below.



(b) Write down the value of k

(1)

(Total for question = 3 marks)



Q8.

The curve **S** has equation $y = f(x)$ where $f(x) = x^2$

The curve **T** has equation $y = g(x)$ where $g(x) = 2x^2 - 12x + 13$

By writing $g(x)$ in the form $a(x - b)^2 - c$, where a , b and c are constants, describe fully a series of transformations that map the curve **S** onto the curve **T**.

.....

.....

.....

(Total for question = 4 marks)



Q9.

The diagram shows part of the curve with equation $y = f(x)$

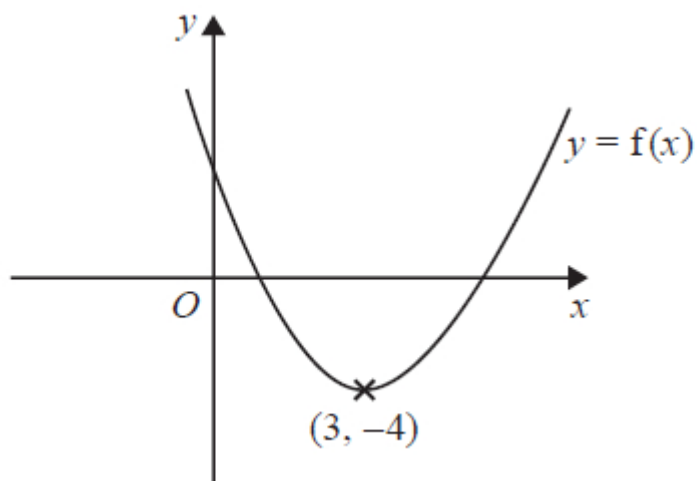


Diagram **NOT**
accurately drawn

The coordinates of the minimum point on this curve are $(3, -4)$

(a) Write down the coordinates of the minimum point on the curve with equation

(i) $y = f(x - 4)$

(..... ,)

(ii) $y = 3f(x)$

(..... ,)

(iii) $y = f\left(\frac{1}{2}x\right)$

(..... ,)

(3)

The curve with equation $y = f(x)$ is translated to give curve C.

C has a minimum at the point with coordinates $(3, 5)$

The equation of C is $y = f(x) + k$

(b) Write down the value of k

$k = \dots\dots\dots$

(1)

(Total for question = 4 marks)



Q10.

A curve has equation $y = f(x)$

The coordinates of the minimum point on this curve are $(-9, 15)$

(a) Write down the coordinates of the minimum point on the curve with equation

(i) $y = f(x + 3)$

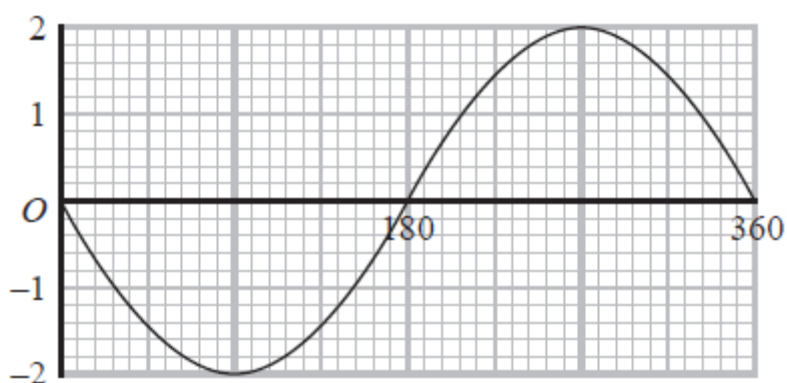
(..... ,)

(ii) $y = \frac{1}{3} f(x)$

(..... ,)

(2)

The graph of $y = a \cos(x + b)^\circ$ for $0 \leq x \leq 360$ is drawn on the grid below.



Given that $a > 0$ and that $0 < b < 360$

(b) find the value of a and the value of b .

$a =$

$b =$

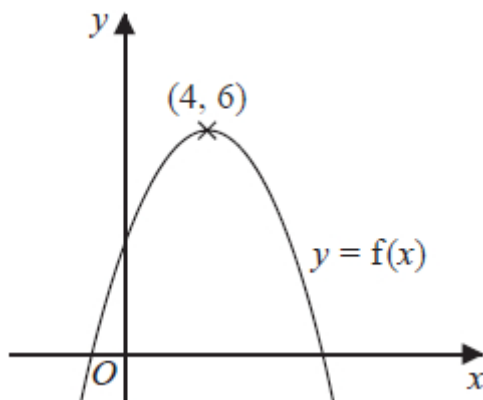
(2)

(Total for question = 4 marks)



Q11.

The diagram shows a sketch of part of the curve with equation $y = f(x)$



There is one maximum point on this curve.
The coordinates of this maximum point are (4, 6)

(a) Write down the coordinates of the maximum point on the curve with equation

(i) $y = f(x + 4)$

(..... ,)

(ii) $y = f(2x)$

(..... ,)

(2)

The equation of a curve **C** is $y = x^2 + 3x + 4$

$$\begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

The curve **C** is transformed to curve **S** under the translation

(b) Find an equation of curve **S**.

You do not need to simplify the equation.

.....

(2)

(Total for question = 4 marks)



Q12.

The curve **C** has equation $y = f(x)$ where $f(x) = 9 - 3(x + 2)^2$

The point **A** is the maximum point on **C**.

(a) Write down the coordinates of **A**.

(..... ,)

(1)

The curve **C** is transformed to the curve **S** by a translation of $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

(b) Find an equation for the curve **S**.

.....

(1)

The curve **C** is transformed to the curve **T**.

The curve **T** has equation $y = 3(x + 2)^2 - 9$

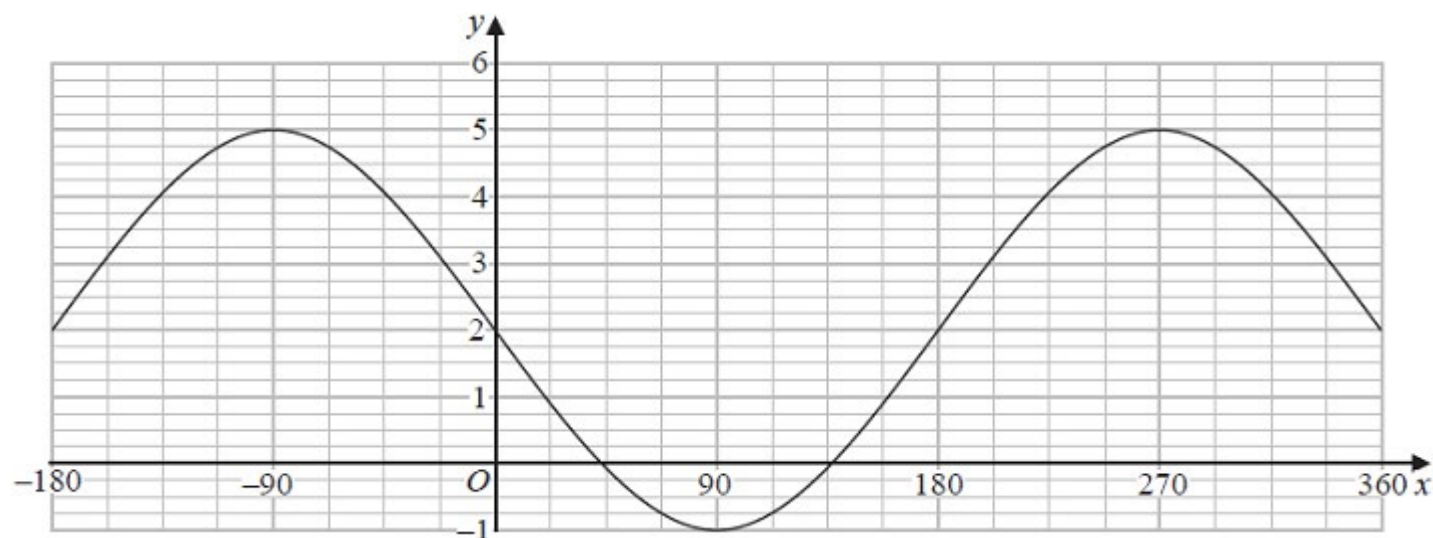
(c) Describe fully the transformation that maps curve **C** onto curve **T**.

.....

(1)



The graph of $y = a \cos (x - b)^\circ + c$ for $-180 \leq x \leq 360$ is drawn on the grid below.



(d) Find the value of a , the value of b and the value of c .

$a = \dots\dots\dots$

$b = \dots\dots\dots$

$c = \dots\dots\dots$

(3)

(Total for question = 6 marks)

Q13.

A particle is moving in a straight line which passes through a fixed point O.

The displacement, s metres, of the particle from O at time t seconds is given by

$$s = 10 + 9t^2 - t^3$$

(a) Find an expression for the velocity, v m/s, of the particle at time t seconds.

$v = \dots\dots\dots$

(2)

(b) Find the time at which the acceleration of the particle is zero.

$\dots\dots\dots$ seconds

(2)

(Total for question = 4 marks)



Q14.

For $y = x^3 - 6x^2 + 20$

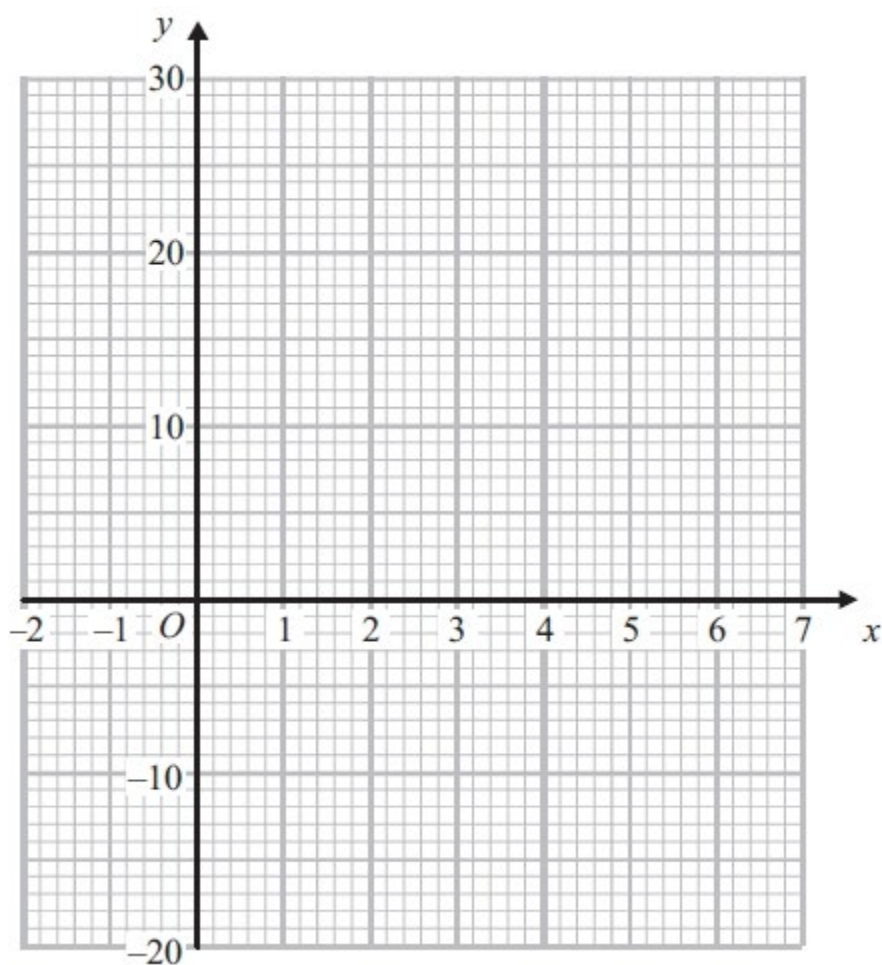
(a) (i) show that $y = 4$ when $x = 2$

(ii) complete the table of values

x	-1	0	1	2	3	4	5	6
y		20	15		-7	-12		20

(2)

(b) On the grid, draw the graph of $y = x^3 - 6x^2 + 20$ for values of x from -1 to 6



(2)



(c) For the curve with equation $y = x^3 - 6x^2 + 20$

(i) find $\frac{dy}{dx}$

.....

(ii) find the gradient of the curve at $x = -3$

.....

(4)

(Total for question = 8 marks)

Q15.

(a) $y = 2x^3 + 3x^2 + 2$

Find $\frac{dy}{dx}$

.....

(2)

(b) The point P lies on the curve with equation $y = 2x^3 + 3x^2 + 2$

The gradient of the curve at P is $-\frac{3}{2}$

Find the coordinates of P .

(..... ,)

(5)

(Total for question = 7 marks)



Q16.

$$y = x^3 + 6x^2 + 5$$

(a) Find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \dots\dots\dots$$

(2)

The curve with equation $y = x^3 + 6x^2 + 5$ has two turning points.

(b) Work out the coordinates of these two turning points.
Show your working clearly.

.....

(4)

(Total for question = 6 marks)



Q17.

For the curve with equation $y = 4x^3 - 2x + 5$

(i) find $\frac{dy}{dx}$

.....

(ii) find the coordinates of the two points on the curve where the gradient of the curve is 1

(..... ,) and (..... ,)

(Total for question = 6 marks)



Q18.

The diagram shows a solid cuboid.

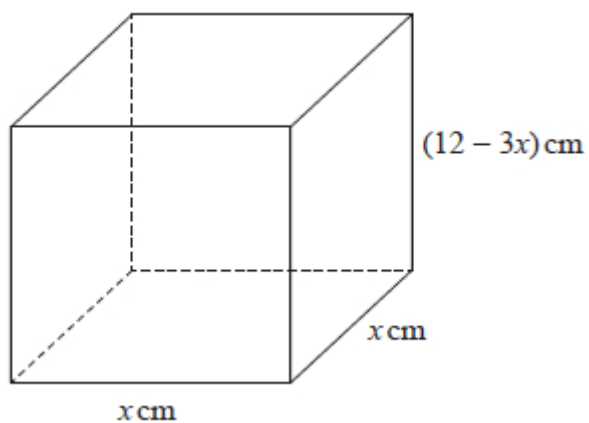


Diagram NOT
accurately drawn

The total surface area of the cuboid is $A \text{ cm}^2$

Find the maximum value of A .

.....

(Total for question = 5 marks)



Q19.

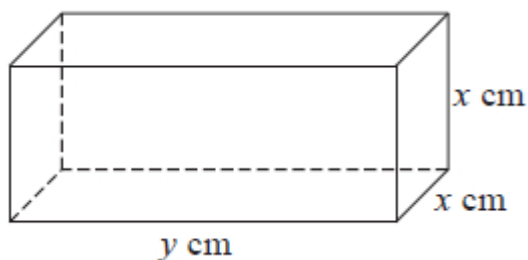


Diagram **NOT**
accurately drawn

The diagram shows a cuboid of volume $V \text{ cm}^3$. The length of the cuboid is $y \text{ cm}$. The width and height of the cuboid are both $x \text{ cm}$.

The total length of all the edges of the cuboid is 112 cm.

(a) Show that $V = 28x^2 - 2x^3$.

(3)

(b) Find $\frac{dV}{dx}$.

$\frac{dV}{dx} = \dots\dots\dots$

(2)

(c) Find the maximum value of V .
Give your answer correct to 3 significant figures.

$V = \dots\dots\dots$

(3)

(Total for question = 8 marks)

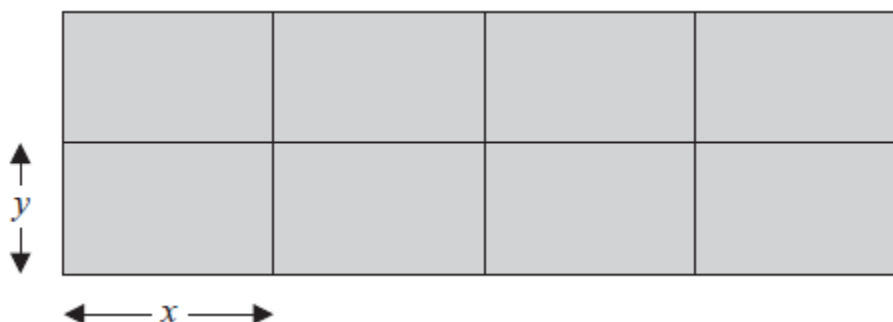


Q20.

A farmer has 180 metres of fencing.

With the 180 metres of fencing, he makes an enclosure divided into eight equal, rectangular pens.

The fencing is used for the perimeter of each pen.



The length of each pen is x metres and the width of each pen is y metres.

(a) (i) Show that $y = 18 - 1.2x$

The total area of the enclosure is $A \text{ m}^2$.

(ii) Show that $A = 144x - 9.6x^2$

(3)

(b) Find $\frac{dA}{dx}$

(2)

(c) Find the maximum value of A .

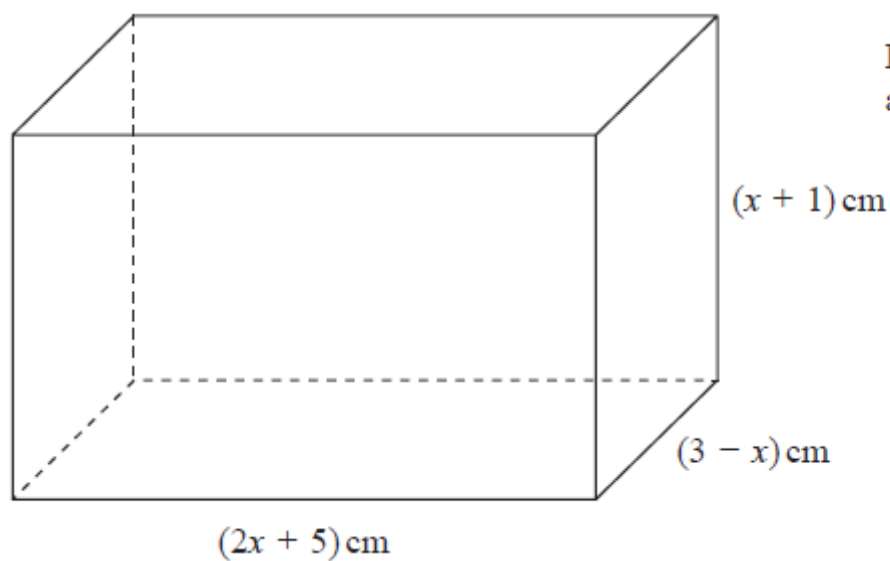
$A = \dots\dots\dots$

(3)

(Total for question = 8 marks)



Q21.



The diagram shows a cuboid of volume $V \text{ cm}^3$

(a) Show that $V = 15 + 16x - x^2 - 2x^3$

(3)

There is a value of x for which the volume of the cuboid is a maximum.

(b) Find this value of x .
Show your working clearly.
Give your answer correct to 3 significant figures.

$x = \dots\dots\dots$

(5)

(Total for question = 8 marks)

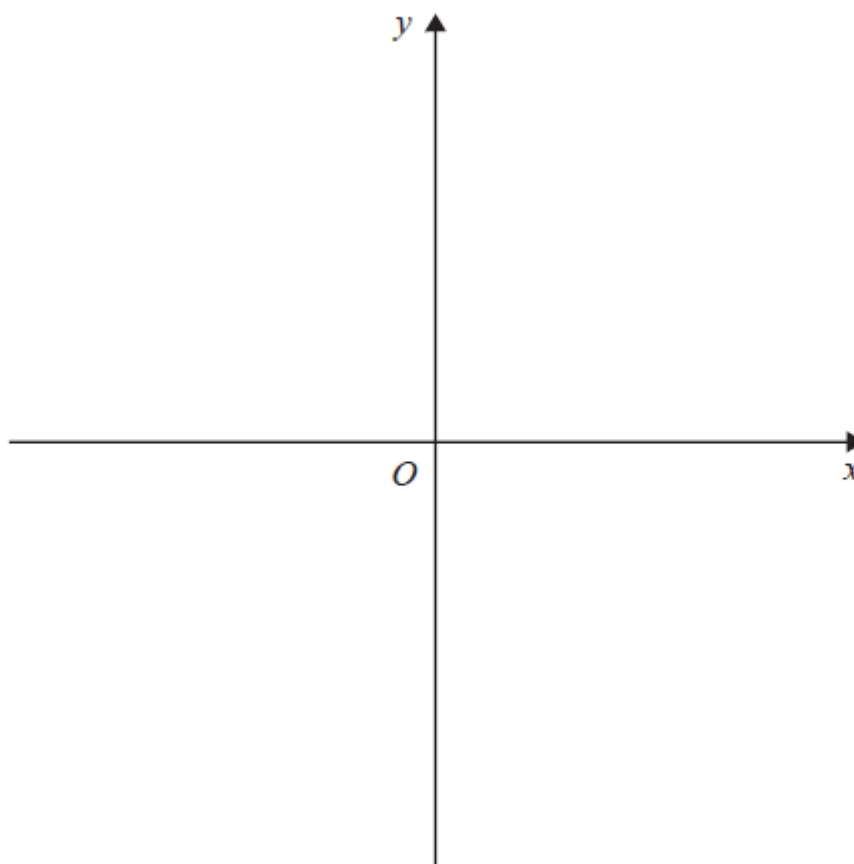


Q22.

The curve C has equation $y = x^2 - 6x + 4$

Using the axes below, sketch the curve C .
On your sketch show clearly

- (i) the exact coordinates of any points of intersection of C with the coordinate axes,
- (ii) the coordinates of the turning point.



(Total for question = 6 marks)



Q23.

The point A is the only stationary point on the curve with equation $y = kx^2 + \frac{16}{x}$ where k is a constant.

Given that the coordinates of A are $\left(\frac{2}{3}, a\right)$

find the value of a .

Show your working clearly.

$a = \dots\dots\dots$

(Total for question = 5 marks)



Mark Scheme

Q1.

Q	Working	Answer	Mark	Notes
(a)		$-\frac{1}{3}$	1	B1 oe allow $-0.\dot{3}$ or -0.33 or better allow $x = -\frac{1}{3}$ or $x \neq -\frac{1}{3}$
(b)	$\frac{2x-3}{3(2x-3)+1}$		2	M1 for substituting $f(x)$ into $g(x)$ Allow $\frac{f}{3f+1}$
	<i>Correct answer scores full marks (unless from obvious incorrect working)</i>	$\frac{2x-3}{6x-8}$		A1 oe (do not isw incorrect cancelling)
(c)	$y(3x+1) = x$ and $3xy + y = x$	or $x(3y+1) = y$ and $3xy + x = y$	3	M1 for moving the denominator to the other side of the equation and expanding correctly
	$x(1-3y) = y$ or $x(3y-1) = -y$	or $y(1-3x) = x$ or $y(3x-1) = -x$		M1 for collecting and factorising the variable on one side in a correct equation
	<i>Correct answer scores full marks (unless from obvious incorrect working)</i>	$\frac{x}{1-3x}$		A1 oe eg $-\frac{x}{3x-1}$ or $\frac{-x}{-1+3x}$ oe
				Total 6 marks



Q2.

Q	Working	Answer	Mark	Notes
(a)	$g(2) = 7 \times 2 - 6 (= 8)$ or $5(7 \times 2 - 6)^2 - 10(7 \times 2 - 6) + 7$		2	M1
	<i>Working not required, so correct answer scores full marks</i>	247		A1
(b)	eg $y = 5(x^2 - 2x) + 7$ or $y = 5(x^2 - 2x + \frac{7}{5})$ oe	eg $x = 5(y^2 - 2y) + 7$ or $x = 5(y^2 - 2y + \frac{7}{5})$	4	M1 or eg $\frac{y-7}{5} = x^2 - 2x$
	eg $y = 5[(x-1)^2 - 1^2] + 7$ or $y = 5\left((x-1)^2 - 1^2 + \frac{7}{5}\right)$ oe	eg $x = 5((y-1)^2 - 1^2) + 7$ or $x = 5\left((y-1)^2 - 1^2 + \frac{7}{5}\right)$ oe		M1 or eg $\frac{y-7}{5} = (x-1)^2 - 1^2$
	$(x-1)^2 = \frac{y-2}{5}$ oe	$(y-1)^2 = \frac{x-2}{5}$ oe		M1 or eg $(x-1)^2 = \frac{y-7}{5} + 1$
	<i>Working not required, so correct answer scores full marks (unless from obvious incorrect working)</i>	$1 + \sqrt{\frac{x-2}{5}}$		A1 Must be in terms of x , oe eg $1 + \sqrt{\frac{x-7}{5}} + 1$ (NB: $f^{-1}(x) = 1 \pm \sqrt{\frac{x-2}{5}}$ is 3 marks)
<i>Alternative for (b)</i>				Total 6 marks
	Let $x = 5y^2 - 10y + 7$ [\Leftrightarrow] $5y^2 - 10y + (7 - x) = 0$ oe		4	M1
	$[y =] \frac{10 \pm \sqrt{100 - 20(7 - x)}}{10}$			M1
	$1 \pm \sqrt{\frac{x-2}{5}}$			M1
	<i>Working not required, so correct answer scores full marks (unless from obvious incorrect working)</i>	$1 + \sqrt{\frac{x-2}{5}}$		A1 Must be in terms of x
				Total 6 marks



Q3.

Question	Working	Answer	Mark	Notes
(a)		$y \geq -3$	1	B1 Accept $g^{-1}(x) \geq -3$
(b)	$(x+3)^2 - 3^2$ or $(x+3)^2 - 9$ or $(y+3)^2 - 3^2$ or $(y+3)^2 - 9$ $y+9 = (x+3)^2$ or $x+9 = (y+3)^2$ $\sqrt{y+9} = x+3$ or $\sqrt{x+9} = y+3$			M1 for completing the square
				M1
				M1
		$-3 + \sqrt{x+9}$	4	A1 oe M3A0 for $-3 + \sqrt{y+9}$ and for $-3 \pm \sqrt{x+9}$

Q4.

Question	Working	Answer	Mark	Notes
(a)		-11	1	B1
(b)		0.5 oe	1	B1
(c)	$g(-1.5) = 1 \div (1 - 2 \times -1.5) (=0.25)$ or $fg(x) = 4 - 3 \times \left(\frac{1}{1-2x} \right)$ oe			M1 $g(-1.5)$ must be the correct calculation alone.
		3.25 oe	2	A1



Q5.

Question	Working	Answer	Mark	Notes
(a)		13	1	B1
(b)	$y = 2(x^2 - 10x) + 9$ or $y = 2\left(x^2 - 10x + \frac{9}{2}\right)$			M1 for a correct equation for a first step in order to complete the square
	e.g. $y = 2((x-5)^2 - 5^2) + 9$ or $y = 2\left((x-5)^2 - 5^2 + \frac{9}{2}\right)$ or $y = 2(x-5)^2 - 41$ oe			M1 dep
	$(x-5)^2 = \frac{y+41}{2}$ oe			M1
		$5 + \sqrt{\frac{x+41}{2}}$	4	A1 oe
				Total 5 marks
Note: Allow candidates to swap x and y when finding the inverse				

Alt	(a)		13	1	B1
	(b)	$2x^2 - 20x + (9 - y) = 0$			M1 for a correct first step
		$x = \frac{20 \pm \sqrt{400 - 8(9 - y)}}{4}$ or $x = \frac{20 + \sqrt{400 - 8(9 - y)}}{4}$			M1 dep
		$x = 5 \pm \sqrt{\frac{41 + y}{2}}$ oe			M1
			$5 + \sqrt{\frac{x + 41}{2}}$	4	A1 oe
					Total 5 marks
Note: Allow candidates to swap x and y when finding the inverse					



Alt	(a)		13	1	B1
	(b)	$2x^2 - 20x + (9 - y) (= 0)$			M1 for a correct first step
		e.g. $2((x-5)^2 - 5^2) + 9 - y (= 0)$ or $2((x-5)^2 - 5^2 + \frac{9}{2}) - y (= 0)$ or $2(x-5)^2 - 41 - y (= 0)$			M1 dep
		$(x-5)^2 = \frac{y+41}{2}$ oe			M1
			$5 + \sqrt{\frac{x+41}{2}}$	4	A1 oe
					Total 5 marks
Note: Allow candidates to swap x and y when finding the inverse					

Q6.

Q	Working	Answer	Mark	Notes
	$y = (x-4)^2 - 4^2 (+5)$ oe or $x = (y-4)^2 - 4^2 (+5)$		3	M1 for a correct first step in order to complete the square
	$y = 4 \pm \sqrt{11+x}$ or $x = 4 \pm \sqrt{11+y}$			A1 allow $y = 4 + \sqrt{11+x}$ or $x = 4 + \sqrt{11+y}$
		$4 - \sqrt{x+11}$		A1 oe
				Total 3 marks



ALT	$x^2 - 8x + (5 - y) = 0$ $(x =) \frac{8 \pm \sqrt{(-8)^2 - 4 \times 1 \times (5 - y)}}{2 \times 1}$ or $y^2 - 8y + (5 - x) = 0$ $(y =) \frac{8 \pm \sqrt{(-8)^2 - 4 \times 1 \times (5 - x)}}{2 \times 1}$		3	M1 for a correct first step in preparation for use of quadratic formula and substitution into the quadratic formula (allow one sign error)
	$y = 4 \pm \sqrt{11 + x}$ or $x = 4 \pm \sqrt{11 + y}$			A1 allow $y = 4 + \sqrt{11 + x}$ or $x = 4 + \sqrt{11 + y}$
		$4 - \sqrt{x + 11}$		A1 oe
				Total 3 marks

ALT	Using $ax^2 + bx + c = a(x + p)^2 + q$			
	$\left(p = \frac{b}{2a}\right) = \frac{-8}{2} (= -4)$ and $q = (4)^2 - 8(4) + 5 (= -11)$		3	M1 for finding p and q
	$y = 4 \pm \sqrt{11 + x}$ or $x = 4 \pm \sqrt{11 + y}$			A1 allow $y = 4 + \sqrt{11 + x}$ or $x = 4 + \sqrt{11 + y}$
		$4 - \sqrt{x + 11}$		A1 oe
				Total 3 marks



Q7.

Question	Working	Answer	Mark	Notes
(a)		Parabola through $(-4,5), (-2,0), (0,-3),$ $(2,-4), (4,-3), (6,0), (8,5)$	2	B2 For a parabola with minimum $(2, -4)$ through at least 5 of $(-4, 5), (-2, 0), (0, -3), (4, -3), (6, 0), (8, 5)$ If not B2 then B1 For u-shaped parabola with minimum $(2, -4)$ or For u-shaped parabola through $(-2, 0), (6, 0)$ or For u-shaped parabola through $(-4, 5), (8, 5)$
(b)		3	1	B1



Q8.

Q	Working	Answer	Mark	Notes
	$[g(x) =] 2(x - 3)^2 - 5$		4	B2 for $a = 2$, $b = 3$ and $c = 5$ correct (stated or shown) (B1 for one of $a = 2$, $b = 3$ and $c = 5$ correct)
	stretch y direction scale factor 2 oe [ft their a] or translation $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$ (ft correct use of their b and c) oe			M1 Stretch and a correct description of the stretch or translation and a correct description of the translation NB: must include the word translation (or translate) and stretch
		Correct transformations in correct order		A1 Stretch y direction scale factor 2 followed by translation $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$ oe eg translation $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$, stretch SF2 in y direction followed by translation $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$
				Total 4 marks
Alt	$[g(x) =] 2(x - 3)^2 - 5$		4	B2 for $a = 2$, $b = 3$ and $c = 5$ correct (stated or shown) (B1 for one of $a = 2$, $b = 3$ and $c = 5$ correct)
	translation $\begin{pmatrix} 3 \\ -2.5 \end{pmatrix}$ (ft correct use of their b and $0.5c$) oe or stretch y direction scale factor 2 (ft their a)			M1 A correct description of the stretch or the translation
		Correct transformations in correct order		A1 Translation $\begin{pmatrix} 3 \\ -2.5 \end{pmatrix}$ oe followed by stretch y direction scale factor 2
				Total 4 marks



Q9.

Q	Working	Answer	Mark	Notes	
(a) (i)		$(7, -4)$	1	B1	
(ii)		$(3, -12)$	1	B1	
(iii)		$(6, -4)$	1	B1	
(b)		9	1	B1	
					Total 4 marks

Q10.

Q	Working	Answer	Mark	Notes
(a)(i)		$(-12, 15)$	1	B1
(ii)		$(-9, 5)$	1	B1
(b)		$a = 2$ and $b = 90$	2	B2 for both values correct (B1 for $a = 2$ or $b = 90$ or $a = -2$ and $b = -90$)
				Total 4 marks



Q11.

Question	Working	Answer	Mark	Notes
(a)(i)		(0, 6)	2	B1
(iii)		(2, 6)		B1
(b)	eg $(x-4)^2 + 3(x-4) + 4$ oe or eg $(x + \frac{3}{2} - 4)^2 - \frac{9}{4} + 4$ oe or eg $x^2 + 3x + 10$ oe or eg $(x + \frac{3}{2})^2 - \frac{9}{4} + 4 + 6$ oe eg $y - 6 = x^2 + 3x + 4$		2	M1 for applying one of the transformations to the equation
		$y = (x-4)^2 + 3(x-4) + 10$ or $y = (x + \frac{3}{2} - 4)^2 - \frac{9}{4} + 4 + 6$		A1 oe eg $y = (x - \frac{5}{2})^2 + \frac{31}{4}$ or $y = x^2 - 5x + 14$ oe
				<i>Total 4 marks</i>



Q12.

Q	Working	Answer	Mark	Notes
(a)		$(-2, 9)$	1	B1
(b)		$(y =) 9 - 3(x - 4 + 2)^2$	1	B1 oe eg $(y =) -3x^2 + 12x - 3$ accept $f(x - 4)$
(c)		Reflection in the line $y = 0$ or x -axis	1	B1 with no mention of another transformation
(d)	$(3, -90, 2)$ $(-3, 90, 2)$ $((3, 270, 2)$ $(-3, 450, 2)$ etc	eg $a = 3$ $b = -90$ $c = 2$	3	B3 for all 3 correct values eg $3, -90, 2$ or $-3, 90, 2$ (If not B3 then B2 for any 2 correct values NB. 2 values from $3, -90, 2$ or 2 values from $-3, 90, 2$ NB: accept a value of $(90 + 360n)$ in place of 90 or $(-90 + 360n)$ in place of -90 where n is an integer (could be negative) If not B2 then B1 for any 1 correct value or the graph of $y = \cos x^\circ$ for $0 \leq x \leq 360$)
				Total 6 marks

Q13.

Question Number	Working	Answer	Mark	Notes
(a)		$(v =) 18t - 3t^2$	2	B2 for $18t - 3t^2$ oe seen as final answer B1 for $18t$ or $3t^2$ or $-3t^2$
(b)		$(a =) "18 - 6t" (=0)$ $(t =) 3$	2	M1ft ft if differentiating correctly a quadratic with 2 or 3 terms A1ft
				Total 4 marks



Q14.

Question Number	Working	Answer	Mark	Notes
(a) (i)	$2^3 - 6 \times 2^2 + 20$		2	B1 must see $8 - 24 + 20$ oe
(ii)		13, (20), (15), 4, (-7), (-12), -5, (20)		B1 for all correct
(b)	Points plotted Curve		1 1	B1 ft $\pm \frac{1}{2}$ square B1 ft if at least B1 scored in (a)
(c) (i)		$3x^2 - 12x$	2	M1 for $3x^2$ or $12x$ A1
(ii)	$3 \times (-3)^2 - 12 \times (-3)$	63	2	M1 ft for a quadratic in c) i) A1
				Total 8 marks

Q15.

Question	Working	Answer	Mark	Notes
(a)		$6x^2 + 6x$	2	B2 Award B1 for any two terms differentiated correctly (remember $\frac{d}{dx}(2) = 0$ is a term)
(b)	$"6x^2 + 6x" = -\frac{3}{2}$		5	M1 Their derivative = -1.5 oe dep on at least B1 in (a)
	$6x^2 + 6x + 1\frac{1}{2} (= 0)$ or $4x^2 + 4x + 1 = (0)$ oe			M1 Correct three terms of quadratic equation on one side.
	$(2x+1)^2 = 0$ or $(x =) \frac{-4 \pm \sqrt{4^2 - 4 \times 4 \times 1}}{2 \times 4}$ or $\frac{-4 \pm \sqrt{16-16}}{8}$ or $\frac{-4 \pm 0}{8}$ or $6((x + \frac{1}{2})^2 - \frac{1}{4} + \frac{3}{12}) (=0)$ oe			M1 Correct factors or correct use of quadratic formula or completing the square.
	$x = -\frac{1}{2}$			A1 For correctly calculating x
		$(-\frac{1}{2}, 2\frac{1}{2})$		A1 For completely correct coordinates.
				Total 7 marks



Q18.

Question		Working		Answer	Mark	Notes
		x^2 oe or $x(12 - 3x)$ oe				M1 for finding an expression for the area of one face
		$x^2 + x^2 + 48x - 12x^2 (= 48x - 10x^2)$				M1 for a complete expression for A (6 sides) with brackets expanded
		$'48 - 20x' = 0$ oe	$'-10'[(x - '2.4')^2 - '2.4'^2]$ oe			M1 for differentiating a correct expression for A (allow 1 error) and equating to zero or completing the square
		$(x = 2.4)$ $48 \times '2.4' - 10 \times '2.4'^2$	$'-10' \times -'2.4'^2$ or $'-10' \times -'5.76'$			M1 ft if previous M1 awarded for isolating x and substituting into A or finding max value of A from completing the square
				57.6	5	A1 accept 58 from correct working
						Total 5 marks



Q19.

Q	Working	Answer	Mark	Notes
(a)	$8x + 4y = 112$		3	M1 correct equation linking x and y
	$y = \frac{112 - 8x}{4}$ oe			M1 expression y in terms of x
	$V = x^2 \times \frac{112 - 8x}{4}$	proof		A1 Conclusion with full working shown
(b)	$56x - 6x^2$		2	B2 B1 for $56x$; B1 for $-6x^2$
(c)	$56x - 6x^2 = 0$		3	M1 fit for equating differential to 0
	$x = \frac{28}{3}$ oe			A1 Correct value for x
			813	A1 for answer that rounds to 813
Total 8 marks				

Q20.

Question	Working	Answer	Mark	Notes
(a) (i)	$12x + 10y = 180$		1	B1 Accept $12x = 180 - 10y$ or $10y = 180 - 12x$
(ii)	$(A =) 4x \times 2y$ $(A =) 4x \times 2(18 - 1.2x)$	proceed to $A = 144x - 9.6x^2$	2	M1 $4x \times 2y$ or $8xy$ oe A1 $4x \times 2(18 - 1.2x)$ or $8x(18 - 1.2x)$ or $4x(36 - 2.4x)$ oe AND proceeding correctly to $A = 144x - 9.6x^2$
(b)		$(dA/dx =) 144 - 19.2x$	2	B2 B1 for 144, B1 for $-19.2x$ Do not isw
(c)	" $144 - 19.2x^2$ " $= 0$ $x = 7.5$ ($y = 9$) $(A =) 144 \times$ " 7.5 " $- 9.6 \times$ " 7.5^2 " or $(A =)$ $8 \times$ " 7.5 " \times " 9 "	540	3	M1 fit Must be a 2 part linear expression M1 dep A1
Total 8 marks				



Q21.

Question	Working	Answer	Mark	Notes
(a)	$(2x + 5)(x + 1) = 2x^2 + 2x + 5x + 5$ $(= 2x^2 + 7x + 5)$ or $(x + 1)(3 - x) = -x^2 + 3x - x + 3$ $(= -x^2 + 2x + 3)$ or $(3 - x)(2x + 5) = -2x^2 + 6x - 5x + 15$ $(= -2x^2 + x + 15)$		3	M1 for multiplying out two brackets correctly at least 3 terms correct M2 for at least 4 terms correct out of a maximum of 8 terms $6x^2 - 2x^3 + 6x - 2x^2 + 15x - 5x^2 + 15 - 5x$
	E.g. $[(2x^2 + 7x + 5)(3 - x) =]$ $-2x^3 - 7x^2 - 5x + 6x^2 + 21x + 15$ or $[(-x^2 + 2x + 3)(2x + 5) =]$ $-2x^3 - 5x^2 + 10x + 4x^2 + 6x + 15$ or $[(-2x^2 + x + 15)(x + 1) =]$ $-2x^3 - 2x^2 + 15x + x^2 + x + 15$			M1 for at least 3 terms correct out of a maximum of 6 terms or for at least 4 terms correct out of a maximum of 8 terms
		Shown		A1



Question	Working	Answer	Mark	Notes
(b)	$\left(\frac{dV}{dx} = \right) 16 - 2x + (3 \times -2x^2)$ oe		5	M1 for the correct differentiation of at least 2 correct terms from 16 or $-2x$ or $(3 \times -2x^2)$
	$\left(\frac{dV}{dx} = \right) 16 - 2x - 6x^2$ oe			A1 for a correct differentiated expression
	' $16 - 2x - 6x^2 = 0$ ' oe			M1 (dep on M1) for equating their differentiated expression to zero
	E.g. $(x =) \frac{-2 \pm \sqrt{2^2 - 4 \times 6 \times -16}}{2 \times 6}$ oe (accept + in place of \pm) or E.g. $6 \left(\left(x + \frac{1}{6} \right)^2 - \left(\frac{1}{6} \right)^2 \right) - 16 (= 0)$ oe			M1 (dep on M1) for a complete method to solve their 3-term quadratic equation (allow one sign error and some simplification – allow as far as $\frac{-2 \pm \sqrt{4 + 384}}{12}$)
		1.47		A1 dep on M1 for answer in range 1.47 – 1.5 from correct working (Must reject -1.80 to -1.81 if calculated)
				Total 8 marks



Q22.

Q	Working	Answer	Mark	Notes	
	e.g. $\frac{dy}{dx} = 2x - 6$ or $(x - 3)^2 - 3^2 + 4$ or $\frac{'3 + \sqrt{5}' + '3 - \sqrt{5}'}{2}$			M1	
	$(x - 3)^2 - 5$ or $x = 3$			A1	(ft providing previous M1 scored) for (3, -5)
	eg. $x = \frac{- -6 \pm \sqrt{(-6)^2 - 4 \times 1 \times 4}}{2 \times 1}$ or $x = 3 \pm \sqrt{3^2 - 4}$ or $(x - 3)^2 = 5$			M1	for correct method to find roots
	$x = 3 \pm \sqrt{5}$			A1	or exact equivalents
		Correct graph	6	B2	for fully correct labelled graph (see end of mark scheme) (B1 for parabola intercepting y-axis at (0, 4))
					Total 6 marks



Q23.

Q	Working	Answer	Mark	Notes
	$\left[\frac{dy}{dx} = 2 \times kx - 16x^{-2} \text{ or } 2kx - \frac{16}{x^2} \text{ oe}\right]$		5	M2 for both terms differentiated correctly (M1) for one term differentiated correctly
	" $2kx - 16x^{-2} = 0$ oe			M1 ft dep on M1
	eg $\frac{8}{27}k = 8$ or $\frac{4}{3}k = 36$ or $k = 27$ oe			M1 (not ft) for substituting $x = \frac{2}{3}$ into their correct equation for k and getting as far as one step from the value of k or the correct value of k
	<i>Working must be seen</i>	36		A1 dep on M4
				Total 5 marks